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XIII. *On a Geometrical Determination of the Conditions of Maximum Efficiency in the case of the Transmission of Power by means of Alternating Electric Currents.* *By* THOMAS H. BLAKESLEY, *M.A.**

1. THIS paper treats the above problem as a particular application of the method of representing Combinations of Electromotive Forces possessing a simple Harmonic Law of Change, already given by the author more than three years ago, and detailed in a series of papers published during 1885 in the 'Electrician.'

It was then pointed out that, though with two constant electromotive forces acting in a simple circuit the transmission efficiency does not exceed the ratio of the smaller to the larger, yet with alternating electromotive forces [denominated by their maximum values] the efficiency may exceed this ratio, owing to the self-induction of the circuit, and to the possibility of varying the interval of time at which the phases of one electromotive force may follow the corresponding phases of the other.

It is here proposed to show how the particular interval of time which will give the maximum efficiency may be geometrically determined, and what the value of that maximum

* Read November 12, 1887.

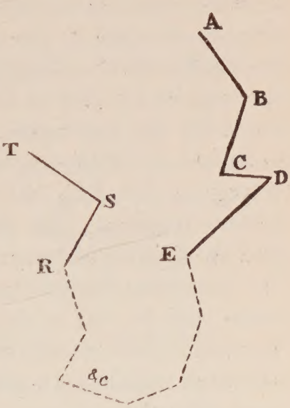
efficiency is for two given electromotive forces undergoing harmonic repetition in the same given period, in a simple circuit possessing a known resistance and a known coefficient of self-induction. The phase-interval is then the only independent variable in the problem, and what its value must be to give the maximum efficiency of transmission of Power, and what that efficiency will then be, are the questions to which we shall have answers from Geometry alone. A short statement of the general method of representation will make the particular steps required for these problems perfectly clear.

2. Let a straight line of fixed length, and situated in the plane of the paper, undergo uniform rotation in that plane. Then its projection upon a fixed indefinitely long line also in that plane will undergo harmonic variation, and may represent any magnitude capable of undergoing such change (*e. g.* an electromotive force), the maximum value of this varying magnitude being represented by the revolving line itself. The period in which the revolving line makes one complete revolution is the period of the change. Hence, if we know the position of the fixed line and of the revolving line at any instant, we can say in what particular phase the magnitude undergoing harmonic change is at that instant. For instance, suppose these lines make 30° with each other, we can say at once that the magnitude is removed from its maximum value by an interval of time equal to one twelfth of the period. If the angle is at the instant increasing, the magnitude has passed its maximum value that interval of time ago. If the angle is growing less, the magnitude will attain its maximum after that interval of time. It is therefore necessary to fix a positive direction of rotation as representing the positive lapse of time. [That direction which is opposite to that of the hands of a watch will here be adopted.]

3. It follows that when we have two such electromotive forces acting in the same circuit, having different maximum values but the same Period, since each is represented by the projection of a revolving straight line upon a fixed straight line, the resultant electromotive force at the instant is the algebraical sum of the individual projections. And if the two revolving lines are laid down as the two sides of a triangle *taken in order*, the rotation being uniform and the same for

both lines, the lines will remain always inclined at the same angle to each other, and the algebraical sum of their projections is the projection of the third side. Thus, in the matter of such electromotive forces, we have a theorem exactly corresponding to the triangle of directed quantities.

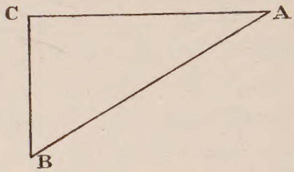
4. We may extend this mode of representing such quantities so as to form a theorem corresponding to the polygon of directed quantities, and cite it thus :—



"If the straight lines AB, BC, CD, . . . ST represent the maximum values of different electromotive forces, and, as to direction, are so laid down upon the paper that their projections upon a fixed straight line represent at some point of time the instantaneous values of those electromotive forces, their instantaneous resultant is the projection of the simple straight line AT."

5. If, in any particular case, we have taken into consideration all the electromotive forces concerned, then clearly the line representing the resultant corresponds in phase with the instantaneous current; and if by scaling or calculation we find the value of this resultant in volts, we have only to divide by the resistance in ohms to obtain the maximum value of the alternating current resulting from all the component electromotive forces. This is true even if one of the electromotive forces is that of self-induction. But suppose we have arrived at a preliminary resultant by compounding all the electromotive forces with the exception of that of self-induction; we then require the final resultant, and we obtain it by remembering that it must be at right angles to the electromotiv force of self-induction; for the electromotive force of self-induction must be greatest when the current is passing through zero: therefore it must have its projection on the fixed line greatest when that of the final resultant (corresponding with the current) is zero. Therefore the final resultant and the electromotive force of self-induction must be to the preliminary

resultant as the two sides of a right-angled triangle, including the right angle, are to the hypothenuse; and as we already possess the hypotheneuse we have only to determine the ratio of the sides, and upon which side of the hypotheneuse they must be placed, in order fully to determine the position and size of the final resultant and the electromotive force of self-induction. The geometrical construction is as follows.



From one end of the preliminary resultant set off an angle in the negative direction of rotation whose tangent is equal to the product of the coefficient of self-induction and the angular velocity of rotation divided by the resistance, and then complete the right-angled triangle. For if ABC is such a triangle,—AB, BC, AC representing respectively the preliminary resultant, the electromotive force of self-induction, and the final resultant at the maximum values,—it is clear that the maximum rate of increase of the resultant electromotive force will be AC × angular velocity. Divide this by the resistance, and the maximum rate of increase in the *current* is obtained, which, multiplied by the coefficient of self-induction, must give the maximum electromotive force of self-induction, from the fundamental conception of that magnitude.

Hence, in symbols, if r = the resistance,

L = the coefficient of self-induction,

ω = the angular velocity,

$$BC = \frac{AC}{r}\omega L,$$

or

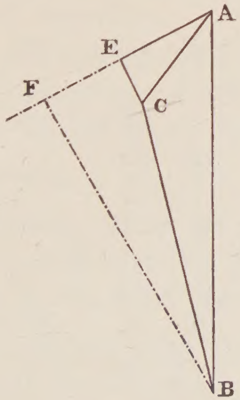
$$\frac{BC}{AC} = \tan BAC = \frac{\omega L}{r}.$$

If 2T is the period, $\omega = \frac{2\pi}{2T}$, $\therefore \tan BAC = \frac{L\pi}{Tr}$.

And since the electromotive force of self-induction must be greatest and +ve when the current is changing *through zero from +ve to −ve*, it is clear that the phases of the electromotive force of self-induction must *follow* those of the final resultant electromotive force at an interval of time represented by a quarter of the period. Thus the above construction is justified.

6. The power working at any instant in a source of electromotive force is the value of the product of the instantaneous electromotive force in question and of the instantaneous current; but this is constantly changing during a period, and the mean power is half the product of the maximum value of the electromotive force, of the maximum value of the current, and of the cosine of the angle representing the time-interval between their similar phases. I have given a geometrical proof of this theorem in "Alternating Currents." It amounts simply to this in the methods of representation here employed, that if we project the revolving line corresponding to any particular source of electromotive force upon the direction of the final resultant, the power derived from this particular source will be the product of such projection and the final resultant divided by twice the resistance. Hence the various powers of the different sources will simply be proportional to the various projections upon the line of the final resultant.

7. Suppose, then, that AB, BC are the revolving representatives of two electromotive forces. Then AC is their resultant; CAE is an angle whose tangent is equal to $\frac{L\pi}{Tr}$, as explained; CE, BF are perpendiculars upon AE. Then AE is the final resultant or effective electromotive force, merely requiring division by the resistance to give the current.



The power derived from the source of AB is AF $\frac{AE}{2r}$; the

power transferred to the source of BC is FE $\frac{AE}{2r}$; and the power heating the circuit is AE $\frac{AE}{2r} = \frac{AE^2}{2r}$.

As regards the projection of BC, viz. FE, since (as here drawn) FE is in a contrary direction to AE, there is a transfer of power to its source. Had F been situated nearer to A than E is, the source of BC would do work and assist in heating the circuit. This obviously depends upon whether BC, AE are inclined to one another at an angle greater or less than a right angle.

If we denominate these three powers as the power of the active source, the power of the recipient source, and the heating-power, they will be to each other in the proportion

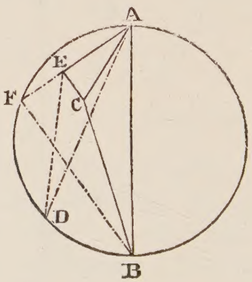
$$AF : FE : AE ;$$

and the efficiency of transmission will be $\frac{FE}{AF}$, the ratio of waste being $\frac{AE}{AF}$.

8. Since AFB is a right angle, F always lies upon the circle described upon AB as diameter. Describe this circle. We may call it briefly the F circle.

At A set off the angle BAD in the negative direction equal to CAE, *i. e.* its tangent is equal to $\frac{L\pi}{Tr}$, and therefore its magnitude is *independent* of the angle of phase-difference $\{\pi - ABC\}$.

Let D be in the F circle, and join DE.



Now it is easy to see that the two triangles EAD, CAB are similar, and their sides homologous in the order of the letters

given. For

$$EA = CA \cos EAC,$$

$$DA = BA \cos DAB = BA \cos EAC,$$

and the angle

$$EAD = EAC + CAD = DAB + CAD = CAB.$$

Hence the third side

$$DE = BC \cos EAC,$$

and therefore is invariable, whatever the phase-difference between the lines AB, BC.

But D is also fixed; therefore E lies upon a circle whose centre is D, and whose radius is BC cos EAC, or BC cos DAB. I call this circle the E circle.

We have now reached the following state of things:—

F must lie upon a fixed circle,

E " " " "

and AEF are in one straight line.

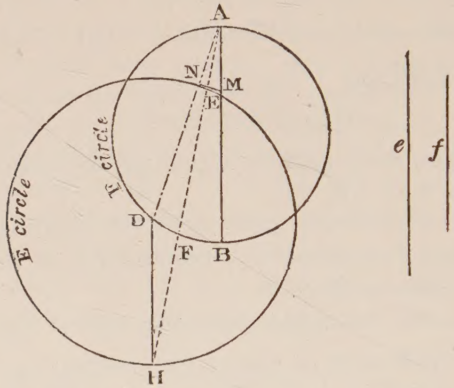
With these prefatory remarks I now proceed to the solution of the questions particularized at the beginning of the paper; which will now be seen to be merely the finding out of the particular position of E upon its circle, which will make the ratio $\frac{FE}{AF}$ as large as possible, or, which is the same thing, that of $\frac{AE}{AF}$ as small as possible.

9. Let e and f be the two electromotive forces. Take AB equal to e, and cut off from it BM equal to f. Describe the F circle upon AB, and set off BAD as before, an angle having its tangent equal to $\frac{L\pi}{Tr}$.

D is on the F circle. Draw MN perpendicular to AD, and with centre D at distance DN describe a circle. This is the E circle, for its centre is at D, and its radius $= DN = BM \cos DAB$.

Through D draw the radius DH parallel to AB, H being on the same side of D that M is of A. Join AH by a straight line cutting the E circle in E and the F circle in F. Then these are the particular positions on the circles which make

the ratio $\frac{FE}{AF}$ a maximum; *i. e.* $\frac{FE}{AF}$, as here found, is the maximum efficiency. And if DE be joined, the angle ADE



will give the phase-difference between the electromotive forces to give this efficiency.

For, in the first place, it is clear that the ratio $\frac{FE}{AF}$ has some maximum (not a minimum) value between the positions where E and F coincide, viz. when the two circles intersect. And because DH is parallel to AB, and AH cuts them, therefore the angle DHE = the angle BAF.

But DH, BA pass through the centres of the two circles respectively; therefore AH, making the same angle with BA as it does with DH, must cut the circumferences of the E and F circles at the same angle. Thus at E and F, the points where AH cuts the E and F circles respectively, the arcs of those circles are parallel. Therefore an elemental displacement of E and F from these positions would result in no alteration of the ratio in which E divides AF.

But this is the characteristic of a maximum or minimum value of such a ratio. Hence the maximum efficiency is $\frac{FE}{AF}$; and it has before been proved that the triangle ADE is similar to one in which the sides homologous to AD, DE are the electromotive forces themselves. Thus, for maximum efficiency, the phases of f must follow the phases of e by an angle equal to $(180° + ADE)$, or, which is the same thing,

precede them by (180° − ADE) or (π − ADE). This, reduced to time, is

$$\frac{(\pi - \mathrm{ADE})}{2\pi} 2\mathrm{T} = \left(1 - \frac{\mathrm{ADE}}{\pi}\right)\mathrm{T}.$$

It is clear that $\frac{\mathrm{DN}}{\mathrm{AD}}$ is the ratio of f to e. The maximum efficiency $\frac{\mathrm{FE}}{\mathrm{AF}}$ exceeds this ; and it is seen to do so in virtue of the existence of a coefficient of self-induction, the absence of which would cause D to coincide with B.

Addendum.—Analytical Expressions.

The value of the maximum efficiency in symbols is

$$\frac{f}{e}\left\{\frac{1 + \frac{f}{e}\cos\beta}{\frac{f}{e} + \cos\beta}\right\},$$

where $\tan\beta = \frac{\mathrm{L}\pi}{\mathrm{T}r}$. The fraction $\frac{1 + \frac{f}{e}\cos\beta}{\frac{f}{e} + \cos\beta}$ can be easily shown to be greater than unity. And the angle ADE may be calculated, if desired, as follows :—

$$\mathrm{ADE} = (2\chi - \beta),$$

where

$$\tan\chi = \frac{\sin\beta}{\cos\beta + \frac{f}{e}}.$$

[χ is the angle DHA or BAH.]

Adapting to logarithms.

Let $$\frac{f}{e} = \cos\alpha,$$

$$\tan\chi = \frac{\sin\beta}{2\cos\frac{\beta+\alpha}{2}\cos\frac{\beta-\alpha}{2}}.$$

There may be transmission of power from the source of e to the source of f even when $f > e$, provided that $f\cos\beta$ is not $> e$; as would appear at once from a geometrical construction on the plan given above, and in any case the condition of maximum efficiency is one of stability.

XIV. *On a Method of Discriminating Real from Accidental Coincidences between the Lines of different Spectra; with some Applications. By* E. F. J. Love, *M.A., Demonstrator of Physics in the Mason College, Birmingham.**

[Plate VI.]

In investigating problems the solution of which depends on the coincidence of the lines belonging to different spectra, *e. g.*, in determining the presence of any particular substance in the Sun's reversing layer, it is usually considered sufficient to demonstrate a close agreement between the wave-lengths of a number of lines in the spectra. That this method, however, taken by itself, does not suffice to give us a reliable result was shown by Schüster†, who demonstrated that, in accordance with the Theory of Probability, a certain number of coincidences between the lines of two spectra might be expected to occur, even if the spectra be quite unrelated; and showed how to calculate the maximum number of coincidences possible on the assumption that no relation exists between them. If no greater number than this is found, the coincidences must be looked upon as probably accidental. The method as given by Schüster is employed to determine whether the lines of a spectrum are harmonically related; but it could obviously be quite as well applied to examine the relations between two different observed spectra. It demands, however, a considerable amount of rather troublesome computation. Further, cases may occur in which approximate equality obtains between a great number of lines in two spectra, one or both of which is so crowded with lines that the question arises, What difference of wave-length between the lines is admissible as a coincidence? This difficulty especially meets us when we are dealing with certain parts of the solar spectrum—especially those obtained from portions of the sun's surface near to, or including, a spot; and Schüster's method gives us no hint towards its solution.

* Read November 26, 1887.

† Proc. Roy. Soc. xxxi. p. 337, 1881.

While engaged in preparing a report on Grünwald's* recent investigations into the relation between wave-length and specific volume, the present writer was led to a simple method of comparison, based on the Law of Error. In accordance with this law, the errors of observation of a single quantity group themselves about the mean value of the quantity in such a way that the number of observations in which the errors are less than some small quantity x is greater than the number in which they lie between x and $2x$, this again is greater than the number between $2x$ and $3x$, and so on; the equation between the number of observations and magnitude of error being, as is well known, of the form

$$y = a\epsilon^{-c^2x^2}.$$

Now since the various spectrum-lines of a substance in a given physical condition are connected by an invariable relation, it seems allowable to assume that observations of the several lines in one spectrum may be regarded as different observations of one phenomenon, viz., that spectrum; as a consequence it is here assumed that, if the differences of the wave-lengths of corresponding lines in spectra really due to the same substance, but determined by different observers, and under different conditions (*e.g.*, the substance as examined by one observer being on the earth; and as examined by the other, on the sun) be compared, they will accord with the Law of Error. The method thence derived is as follows:—The differences between the wave-lengths of the lines compared are arranged in groups, each group containing those observations the errors of which lie within certain narrow limits. The number of observations in each group is then plotted as an ordinate of a curve, the average error of the group being the abscissa. If this curve be then compared with the curve given by the Law of Error, any serious divergence from the form of the latter curve is at once made manifest. It should, however, be borne in mind that the Law of Error admits the possibility of errors of every conceivable magnitude, and assumes the number of sources of error to be practically infinite; as a result we should expect the curves actually obtained to be steeper if anything than the theoretical curve in the portion near the y-axis.

* *Astr. Nachr.* Bd. 117; Phil. Mag. [5] xxiv. p. 354, 1887.

It is obvious that the method can only be applied to spectra which contain a considerable number of lines, and that measurements of *all* the coincidences observed must be included, otherwise the method will not give a correct result; it might under different circumstances cause us either to underrate or overrate the probability in favour of the coincidence. On the other hand, in dealing with the solar spectrum with the aid of Ångström's map, it must be borne in mind that the map is very incomplete, many lines being omitted; as a result we must not expect to find *all* the lines of any substance in the map, even if these lines exist in the sun.

We may illustrate the method by means of the curves shown in figs. 1–5 (Plate VI.). Fig. 1 is the Theoretical Curve of Error; fig. 2 that actually obtained by comparing the values of 21 lines in the arc-spectrum of iron, observed by Ångström, with their values obtained by Cornu; fig. 3 is the curve obtained on comparing Kirchhoff's measurements of the spark-spectrum of cerium (27 lines) with those of Thalén. The agreement in form between these curves is obvious, and, considering the somewhat small number of lines included, rather striking; the greater steepness of the two latter curves (owing to the finite number of sources of error) is also well marked. Fig. 4 contains a comparison of the arc-spectra of iron and nickel (19 lines) between wave-lengths 4850 and 5890; and fig. 5 a comparison of those of iron and titanium (34 lines) between the same limits. The divergence of these from fig. 1 is, as we should naturally expect, very marked; so far as *any* curves may be said to represent the results, the best would be straight lines nearly parallel to the x-axis.

Having given these instances as illustrations of the degree of accuracy of the results to be obtained from the method, we will proceed to examine its bearing on two problems of considerable interest: (1) The existence of cerium in the sun; (2) Professor Grünwald's recent investigations.

Cerium in the Sun.

The existence of cerium in the sun's reversing layer was indicated as probable by Professors Liveing and Dewar* in 1882; and in the following year the same observers† pub-

* Proc. Roy. Soc. xxxiii. p. 428, 1882

† Phil. Mag. [5] xvi. p. 401, 1883.

lished a map, extending from F to *b*, showing among others the arc-spectrum of cerium, and the spectra of the lines seen widened in two sun-spots at Greenwich in 1881. The spectrum of cerium exhibits numerous coincidences with widened lines, though comparatively few with lines given in Ångström's map. On the other hand, Messrs. Hutchins and Holden*, who have re-examined the evidence for the existence of certain substances in the sun, with the aid of photography, remark, 'So numerous are the lines [of cerium, molybdenum, uranium, and vanadium] that often on the photographs the total space occupied by them is greater than the space not so occupied. . . Evidently coincidences between these and solar lines cannot fail to occur as matters of chance and therefore prove nothing. One can easily count a hundred or so such coincidences without the slightest conviction that the connexion is other than fortuitous."

As a complete map of the arc-spectrum of cerium has not yet been published, the writer has been obliged to fall back on the evidence given by Liveing and Dewar's map, mentioned above. It is briefly as follows :—Of the 34 lines included in the map 20 coincide with solar lines which are not already assigned or possibly assignable to other metals; of these solar lines 6 only are represented on Ångström's map, and *the whole* have been observed to be widened, most of them very considerably, in sun-spots, though not all in the same spot. This fact by itself considerably strengthens the case for the existence of cerium as a constituent of the sun, since it demonstrates a connexion between these lines. Let us apply the method to these twenty coincidences. The differences between the wave-lengths of the cerium and solar lines are given in the following table :—

Between	0·0	and	0·1	Xth metre,		12	coincidences.
,,	0·1	,,	0·2	,,	,,	2	,,
,,	0·2	,,	0·3	,,	,,	2	,,
,,	0·3	,,	0·4	,,	,,	1	,,
,,	0·4	,,	0·5	,,	,,	0	,,
,,	0·5	,,	0·6	,,	,,	2	,,
,,	0·6	,,	0·7	,,	,,	1	,,

* Proc. Amer. Acad. Arts and Sci. xxiii.; Phil. Mag. [5] xxiv. p. 325, 1887.

These numbers when plotted give the curve represented in fig. 6. This curve closely resembles figs. 1, 2, and 3, except in respect of its greater steepness; the latter, so far from being an objection, testifies to the fewness of the sources of error affecting the wave-length measurements. The evidence afforded by the method appears then to supply a substantial confirmation to the reality of the coincidences, so far as the material for investigation goes.

Professor Grünwald's Investigations.

The most important verification of his theory put forward by Grünwald is that afforded by the close agreement between the wave-lengths of the lines in the spectrum of water, as deduced by him from those of the hydrogen spectrum, and their values as obtained by observation. So far, 58 of the predicted lines have been observed by Prof. Liveing; and the remainder occur in a part of the spectrum not yet submitted to examination. The wave-lengths are given in Grünwald's* paper; the differences between the observed and calculated values are given here:—

Between	0·0	and	0·1	Xth metre,		7	coincidences.
,,	0·1	,,	0·2	,,	,,	10	,,
,,	0·2	,,	0·3	,,	,,	8	,,
,,	0·3	,,	0·4	,,	,,	7	,,
,,	0·4	,,	0·5	,,	,,	2	,,
,,	0·5	,,	0·6	,,	,,	2	,,
,,	0·6	,,	0·7	,,	,,	5	,,
,,	0·7	,,	0·8	,,	,,	4	,,
,,	0·8	,,	0·9	,,	,,	5	,,
,,	0·9	,,	1·0	,,	,,	2	,,
,,	1·0	,,	1·1	,,	,,	2	,,
,,	1·1	,,	1·2	,,	,,	1	,,
	...		..			...	
,,	1·5	,,	1·6	,,	,,	1	,,

These when plotted give the curve fig. 7, which agrees almost exactly with the formula for the Theoretical Curve of Error, as tested by taking points. On the other hand, four of the points do not agree so well with the smooth curve as those in

* Phil. Mag. [5] xxiv. p. 357, 1887.

the cases already examined; and until further measurements are obtained, the method only warrants us in asserting that the balance of probability lies on the side of the reality of the coincidences.

The curve shown in fig. 8 is extremely interesting; it is plotted from the comparison, in Grünwald's paper, between the spectrum of one of his hypothetical constituents of hydrogen, termed by him "*b*," and the nearest solar lines, as given in Ångström's map. The number of lines included is 41. The differences are as follows:—

Between	0·0	and	0·1	Xth	metre,	8	coincidences.
„	0·1	„	0·2	„	„	9	„
„	0·2	„	0·3	„	„	7	„
„	0·3	„	0·4	„	„	2	„
„	0·4	„	0·5	„	„	8	„
„	0·5	„	0·6	„	„	4	„
„	0·6	„	0·7	„	„	1	„
„	0·7	„	0·8	„	„	0	„
„	0·8	„	0·9	„	„	1	„
„	0·9	„	1·0	„	„	1	„

Here again two points are very much out; but an over-estimation of $\frac{1}{10}$th of a Xth metre (a quite possible mistake) in reading off Ångström's scale at three different places would replace these two points in the curve; we may therefore fairly consider that the method affords support to Grünwald's hypotheses.

A curious point in connexion with figs. 7 and 8 lies in the fact that in both the first experimental ordinate, instead of being the greatest, is smaller than the second. Is it possible that this may indicate a systematic error in Grünwald's calculations? The probability of this seems increased by an examination of the errors with regard to sign; for, in the comparison of the hydrogen and water spectra, the average positive error (obtained by dividing the sum of all the errors in which the predicted exceeds the observed value by the number of such errors) is $\frac{13\cdot 6}{22} = 0\cdot 6$ Xth metre; while the average negative error is $\frac{17\cdot 2}{33} = 0\cdot 5$ Xth metre. Since a

constant arithmetical error is highly improbable, the only explanation of this seems to be a small systematic difference between Hasselberg's scale of measurement for the hydrogen-spectrum (from which Grünwald's water lines are obtained by halving the wave-lengths of the hydrogen-lines) and Ångström's scale. As it would seem to be very difficult to detect such a difference of scale in any other way, this example adds another to the purposes to which this method may be applied.

XV. *The Effect produced on the Thermoelectrical Properties of Iron when under Stress or Strain by raising the Temperature to Bright-red.* *By* HERBERT TOMLINSON, *B.A.**

THE author has already had the pleasure of communicating to the Physical Society † the results of some experiments relating to a certain curious behaviour of iron wire when under stress or strain at a temperature of bright red. It was found that if the metal was under a slight stress either of torsion or of flexure, there was *apparently* a sudden increase of elasticity when, on heating, the critical temperature had been reached; that is to say, if the wire was under a slight twisting stress it suddenly untwisted, and if under a slight bending stress it suddenly unbent. Corresponding sudden changes in the opposite direction occurred on cooling. When, however, the wire was permanently strained and relieved from stress the equally sudden changes which ensued at the critical temperature were opposite in direction to the changes produced when the wire was under stress; namely, a permanently twisted wire free from torsional stress would suddenly twist further when, on heating, a temperature of bright red was reached, and a permanently bent wire would suddenly bend further. Likewise the sudden changes which took place on cooling were in opposite directions for stress and permanent strain: the permanently twisted wire suddenly untwisted, and the permanently bent wire unbent.

Those sudden changes were regarded as probably indicating

* Read November 26, 1887.

† *Ante*, p. 67.

equally sudden changes in the molecular architecture of iron; and experimental evidence was brought forward to prove that the temperature at which the above-mentioned phenomena occurred was not the same as that at which iron suddenly loses its magnetic properties.

Again, Sir W. Thomson* arrived at the remarkable conclusion that when a permanent strain is left after the withdrawal of the stress producing it, the residual *thermoelectrical* effect is the reverse of the thermoelectrical effect which is induced by the stress and which subsists as long as the stress acts.

The author was led by the above-mentioned considerations to attempt to ascertain how far the thermoelectrical properties of iron when under temporary stress or permanent strain might be affected by raising the temperature to a bright red.

Experiment I.

A piece of well-annealed iron wire one millimetre in diameter was subjected throughout half its length to a great many turns of permanent torsion. It was then supported on the ring of a retort-stand, being insulated from the latter by paper, with the junction of the twisted and untwisted portions in the centre of the ring. The free ends of the iron wire were connected by silk-covered copper wire with the terminals of a delicate reflecting-galvanometer of about 7 B.A. units resistance; the two junctions between the iron and copper being tied together and well wrapped up in tissue-paper, which served also to insulate them from each other. The junction of the twisted and untwisted portions of the iron wire was now heated slightly by a Bunsen's burner; this caused a deflection of the galvanometer, indicating a current from the strained to the unstrained portion through the hot junction, as already found by Sir William Thomson. The flame of the burner, which was placed underneath the junction, was at first kept small, but was afterwards increased by degrees. At about dull red temperature the deflection nearly ceased to increase, but as soon as the wire assumed a bright red colour, the light reflected from the mirror of the

* "Electro-dynamic Qualities of Metals," Phil. Trans. Part IV. 1856.

galvanometer went off the scale. The adjusting magnet was therefore used to bring the spot of light back on to the scale in the hope of getting a fairly steady deflection; but this was found to be impossible, the light frequently changing its position by fits and starts and sometimes going over to the other side of the scale. The reason of this unsteadiness will be explained later on.

The experiment was repeated with a second piece of the same wire similarly treated, with this difference, however, that after slightly heating the junction to observe the direction of the deflection, a large resistance was introduced into the galvanometer circuit. By this means the variations of the sudden deflection again produced at the temperature of bright red were much diminished, and a moment of fair steadiness was seized to remove the burner and allow the wire to cool. Almost immediately after the removal of the burner the light on the scale, *after pausing slightly*, went suddenly in the opposite direction.

The experiment was repeated some six or seven times with different pieces of the same wire, but always with like results.

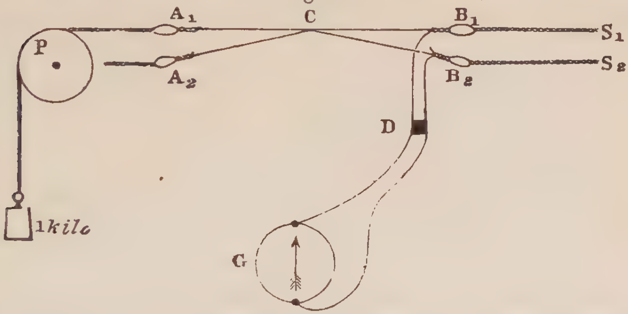
Experiments similar to the above were tried with other pieces of the same iron wire, to which permanent strain was imparted by hammering transversely, by stretching longitudinally, or by bending the wire into coils of small diameter. In all these experiments the thermoelectrical currents produced by moderate heating were, as Sir W. Thomson has already found, from the strained to the unstrained through the hot junction, and in all there was a very sudden increase of the current when, on being heated, the wire became bright red, and an equally sudden decrease when the wire was cooled.

Experiment II.

Two pieces of the same wire were arranged as in fig. 1. $A_1 B_1$, $A_2 B_2$ are two wires brazed together at C and looped at A_1,B_1 and A_2, B_2; from B_1 and B_2 the wires continue to D, where they are joined to the connecting wires of the galvanometer G; D was well wrapped up in tissue-paper. $B_1 S_1$, $B_2 S_2$ are pieces of strong string fastened at S_1, S_2 to the ring of a retort-stand weighted at the bottom. From A_1 passes

a string over a pulley P, and to the end of this string a weight of 1 kilo. was attached; from A_2 passes another string

Fig. 1.



to a support so as to hold up the portion A_2C of the wire A_2, B_2. The two wires A_1 B_1, A_2 B_2 lie in the same horizontal plane; to the left of C they are covered with sand from C to a distance of 10 centims. To the right of C for a distance of about 3 centims. the wires are bare, but beyond this for about 7 centims. are covered with sand; the sand is sustained on two stands placed to the right and left of C.

First, the junction C was slightly heated by a burner, and as the weight of 1 kilo. was not sufficient for the moderate temperature then used to produce any sensible thermoelectrical effect, the end A_1 was pulled rather hard by the hand to ascertain what would be the direction of the thermoelectrical current due to the stress; this direction was found to be from the unstressed to the stressed wire through the hot junction, as Thomson had already found. A rather large resistance was then introduced into the galvanometer circuit for the purpose of diminishing the sensibility of the instrument, and the parts of the uncovered wires to the right of C were raised to the temperature of bright red. Immediately this temperature was reached, there was a sudden start of the galvanometer-needles, indicating that the stretched wire had suddenly become more *negative* relatively to the unstretched wire. When the wires were allowed to cool there was, after a short pause, an equally sudden deflection in the opposite direction.

The weight of 1 kilo. was now supported by blocks placed underneath it, and the junction C was again heated to a bright red. At the critical temperature there was again a

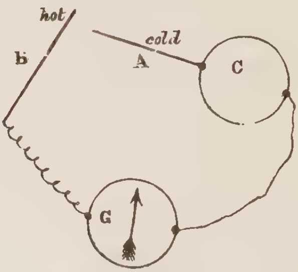
sudden deflection, now indicating that the wire, which had been previously permanently lengthened by the 1 kilo. weight at the bright red temperature, was rendered suddenly more *positive* relatively to the unstrained wire. When the wires were allowed to cool there was, after a short pause, an equally sudden deflection in the opposite direction. This experiment was repeated with three different pairs of wires and with the same results.

It has been mentioned in Experiment I. that as soon as the temperature reached a bright red the deflection of the galvanometer frequently altered by fits and starts, and was even at times strongly reversed in direction. This circumstance constituted a difficulty in all the experiments, and was presently found to be due to slight shiftings of the flame of the burner from slight draughts in the room. Hot iron at any temperature is thermoelectrically negative to cold iron; but when the temperature reaches a bright red there is such a sudden accession of negativeness that if we lay a piece of iron at this temperature upon another piece of cool iron there is developed an electromotive force which, thermoelectrically considered, is very large.

Experiment III.

In fig. 2 A is a piece of the same iron wire connected with one terminal of a condenser C having a capacity of one third

Fig. 2.



of a microfarad; B is a second piece of the same iron wire connected by insulated copper wire with one terminal of a reflecting-galvanometer G having a resistance of about 6,000 B.A. units; the other terminals of the condenser and galvano-

meter are connected together with insulated copper wire. The free end of B was heated in a burner nearly to a white heat, and then, having first been removed from the flame, was quickly placed on the free end of A; immediately there was a deflection of the spot of light on the galvanometer-scale through 10 divisions, showing that the condenser was being charged. Almost immediately after B had touched A, it was removed, and after allowing sufficient time for the wires to cool B was again placed on A, when a deflection through 10 divisions of the scale in the opposite direction indicated the discharge of the condenser. This experiment was repeated several times with fairly uniform results; and by comparing the deflection thus obtained with that produced by a Daniell's cell, it was ascertained that the E.M.F. produced by putting iron at a bright red temperature on iron at about 15° C. was one twentieth of a volt, *i. e.* considerably more than twice as much as would be developed in a single element of bismuth and antimony with one junction at 100° C. and the other at 0° C.

It has long been known that hot iron is negative to cold iron, and this seems to be so at all temperatures, but it has not, the author believes, been noticed before* how very suddenly the negativeness is increased when the temperature reaches a bright red. If the wire B be heated to a temperature between dull red and bright red before being placed on A, there is a current from A to B which can be readily detected if the condenser be cut out and the galvanometer connected directly with A and B, but no effect whatever could be detected when the condenser was used as in the experiment.

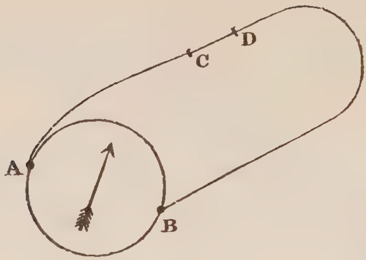
Experiment IV.

A piece of the same iron wire was connected directly with the two terminals A, B of a reflecting galvanometer of 7 B.A. units resistance (fig. 3). A portion C D of the wire was heated above a bright red by a burner, and as soon as the required temperature had been reached the burner was shifted suddenly and slightly towards C; almost immediately

* Unless we except Prof. P. G. Tait's observations, which will be referred to presently.

afterwards the light darted off the scale of the galvanometer, indicating a strong current from A to D through C. The

Fig. 3.



portion C D was again heated above a bright red temperature, and the burner then was shifted suddenly towards D; the deflection of the galvanometer-needle was now strong in the opposite direction, showing a current from B to C through D.

Experiment III. furnishes the clue to the explanation of the phenomenon described in Experiment IV. The portion C D, when at a bright red, is thermoelectrically negative to the rest of the iron wire, and is in fact like a different metal. So long, however, as the flame of the burner is kept in one position the temperature at C will be the same as at D, and there will be no current; but directly we shift the burner towards C we make C hotter than before, whilst D cools more or less, and we get a current from A to D through C.

We also see why a slight draught of air blowing the flame of the burner towards C or D may suddenly generate a strong current in one direction or the other and may completely mask the phenomena recounted in Experiments I. and II.

Experiment IV. may be varied in several ways, thus:—If we keep the flame steadily burning so as to maintain C D at a high temperature, and allow a small stream of water to flow down on C, we get a continuous current from B to C through D, and if we shift the stream to D we reverse the current. Or, instead of the stream of water, we may blow with a small pair of bellows at C or D.

What connexion there may be between the above-mentioned phenomena and the discovery of Professor P. G. Tait that the thermoelectrical diagram of iron cuts that of

iridium-platinum at least *three times* below a low white heat*, that is to say that an iron and iridium-platinum circuit has at least *three* neutral points, does not quite appear.

In conclusion, it may be said that the investigation serves still further to confirm the author in his opinion that a sudden and profound change takes place in the molecular arrangement of iron at a temperature of bright red.

XVI. *The Recalescence of Iron.* By HERBERT TOMLINSON, *B.A.*†

Introduction.

MANY specimens of iron and steel, when they have been raised to a white heat and are cooling, exhibit at a certain stage a remarkable phenomenon—the metal, to all appearances, receives a sudden accession of heat, and reglows. This phenomenon was discovered by Professor Barrett, and is frequently designated the "recalescence" of iron. The author believes the recalescence of iron to be due to a sudden *physical* change, the event of which has been retarded by what he will call "subpermanent retentivity." The retentivity of a substance may be defined to be that property by virtue of which the substance does not *immediately* recover from the strain produced by a stress when the stress has been removed. If the original strain produced by the stress be large, the residual strain will consist of two parts: one permanent, which will not disappear even after the lapse of any length of time; the other subpermanent, which will disappear after a greater or less interval of time, and which may be aided in doing so by molecular vibrations set up by mechanical or any physical agency. Subpermanent retentivity appears with a great variety of strains:—when a wire is twisted or bent, when a piece of iron or steel is magnetized, when a Leyden jar is charged, and when glass has been raised

* Heat, by P. G. Tait, § 199. Also Trans. R. S. E. 1873, for similar peculiarities in nickel at lower temperatures. The author has failed to observe in nickel wire any such phenomena as those mentioned in Experiments I. and II.

† Read December 10, 1887.

in temperature. The last is an example of subpermanent strain resulting from thermal agency, and is the strain with which we have now to deal. Such a strain is not peculiar to glass; for the author has found that a coil of zinc wire, which had been previously heated to nearly the melting-point of the metal and afterwards suddenly cooled by plunging into cold water, continued to show decrease of volume for some minutes after the cooling. The internal friction of a metal seems to be intimately connected with the subpermanent retentivity; and the author has already investigated the internal friction of several metals at various temperatures*. The results of the above-mentioned investigation show that changes of temperature, even when they do not exceed 100° C., exercise a very considerable effect on the internal friction; and it seemed likely that a preliminary examination of the internal friction of iron at very high temperatures might be of service in elucidating the problem of recalescence.

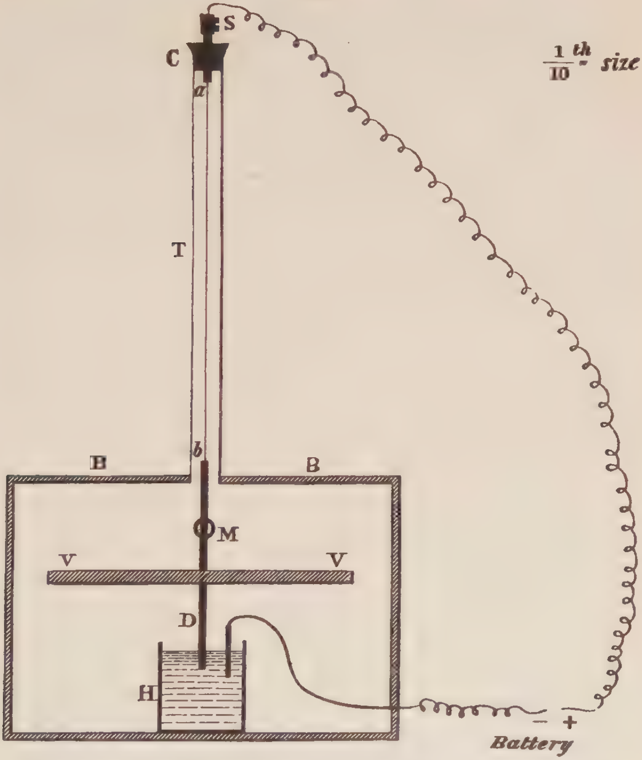
The Internal Friction of Iron at High Temperatures.

Experiment I.—In fig. 1, *a b* is an iron wire 49·5 centim. in length and ·041 centim. in diameter; the ends *a* and *b* are brazed to two pieces of rather stout brass wire; the upper piece of brass wire is furnished with a binding-screw, S, and the lower one is soldered to the centre of a hollow horizontal brass bar, V; D is a piece of copper wire soldered at its upper extremity to V, and dipping with its lower extremity into mercury contained in a glass vessel, H. The wire hangs in the axis of a glass tube, T, fitting into the cover of a box, B; the tube is closed with a cork, C, and the box B, though provided with a door which can be opened for making adjustments or for starting the torsional oscillations, is kept shut up during the actual observations. The box is also provided with a glass window, which enables an illuminated circle crossed by a fine vertical dark line to be thrown on to the light mirror, M, and hence be reflected back on to a scale according to the usual arrangement. The wire was set in torsional oscillation by means of a light feather gently pressed

* Phil. Trans. vol. clxxvii. (part ii. 1886) p. 801. Proc. Roy. Soc. No. 244 (1886), p. 343.

against one end of V and then removed*; the amplitudes of the vibrations were never allowed to exceed one degree.

Fig. 1.



The wire was heated by means of an electric current from a battery of 30 Grove cells, an ampere-meter and a set of resistances serving respectively to measure and to vary the strength of the current. The internal friction of the iron was measured by the logarithmic decrement of arc of the wire when set in torsional oscillation. Part of the whole observed decrement is due to the friction of the air, and part to the friction of the wire D against the mercury in H. The former of these can be calculated from formulæ given in the author's

* It would have been better to have had a small piece of iron attached to V and to have started the vibrations by a magnet on the outside of the box; since the amplitudes diminished very rapidly at the higher temperature.

paper on the internal friction of metals*; the latter by observing the logarithmic decrement at the temperature of the room, first when D dips into the mercury, and again when it does not, whilst from this again the effect of the friction of the mercury at higher temperatures can be approximately determined provided the change wrought in the torsional elasticity be known. The logarithmic decrement resulting from the friction of the air and the friction of the mercury conjointly was small compared with the whole observed decrement, and was in all the calculations allowed for.

The logarithmic decrement due to internal friction was determined in the first instance before the wire had been heated at all, and proved to be

·002203†.

The wire was now heated to the highest temperature recorded in this experiment, and was maintained for about half an hour at this temperature, being all the time kept in torsional oscillation; it was then allowed to cool, and immediately after cooling was again tested. The logarithmic decrement now proved to be ·019367.

After a rest of one hour the logarithmic decrement had considerably diminished, and finally fell to

·00977.

Lastly, the wire was again raised to the previous high temperature, and as soon as the logarithmic decrement and vibration-period had become sensibly constant‡ their values were registered. The temperature was then lowered by putting resistance into the battery-circuit, and a fresh set of observations was taken, and so on, until eventually the temperature of the wire was reduced to that of the room, when the logarithmic decrement proved to be sensibly the same as the one last recorded. The results of the observations, which

* Phil. Trans. vol. clxxvii. (part ii. 1886) pp. 813–815.

† The wire was unannealed, otherwise the logarithmic decrement would have been probably less than half of the amount here given.

‡ Any change of temperature, whether in the direction of increase or decrease, is always attended in the first instance with a larger internal friction than that which ultimately prevails when the vibrations are continued long enough.

extended over a period of several days, are given in the following table :—

Time of vibration, in seconds.	Modulus of torsional elasticity, in grammes per square centim.	Temperature, in degrees Centigrade, t.	Logarithmic decrement due to internal friction, λ.	Difference between consecutive temperatures, Δt.	Difference between consecutive values of λ, $\Delta\lambda$.	$\Delta\lambda : \Delta t$.
1·523	$726{\cdot}6\times10^6$	20	·00977			
1·573	681·1	220	·01805	200	·00828	·000041
1·700	583·2	543	·05033	323	·03228	·000100
2·020	413·1	930	·09634	387	·04601	·000119
2·100	382·2	989	·11216	59	·01582	·000268
2·486	272·7	1181	·20950	192	·09734	·000507
2·600	249·4	1220	·19680	39		

The temperatures given in the third column of the table are deduced from the values of the modulus of torsional elasticity, given in the second column, by means of the formula

$$z_t = z_0(1 - {\cdot}0002442\,t - {\cdot}0000002510\,t^2)^*,$$

where z_t and z_0 are the values of the modulus at t° C. and 0° C. respectively. This formula was itself deduced from some very careful observations of the torsional elasticity of iron at different temperatures between 0° C. and 100° C.; but the numbers given in the third column must only be regarded as rough approximations to the true values; inasmuch as in the first place the above formula was calculated from observations made with a different specimen of annealed iron, and in the second it by no means follows that the formula would apply for such high temperatures as are here recorded. It would seem, however, from the last column of the table, that somewhere near 550° C. there begins to be a rapid rise in the internal friction of the iron, and another still more rapid rise at a temperature of about 1000° C. At this last temperature the internal friction becomes so great that only two or three vibrations are made by the wire before it comes practically to rest; and it is necessary to multiply the observations very considerably before any approach to accuracy in the results

* Proc. Roy. Soc. No. 244 (1886), p. 343.

can be attained. Some idea of the enormous friction encountered by the molecules may be arrived at by bearing in mind that the logarithmic decrement of a torsionally vibrating iron wire at 1000° C. is *ten times* the logarithmic decrement of a tin wire at a temperature of 20° C., tin possessing the largest internal friction of any metal yet examined at ordinary temperatures.

From the last line in the table it would seem that the internal friction of iron begins to decrease as the temperature rises from 1100° C. to 1200° C.; but the numbers here given must be received with some caution in so far as they are to be used for determining this point*.

We tread on surer ground when we compare the second and fourth columns together; and it is abundantly evident that the decrease of torsional elasticity which ensues on rise of temperature bears no comparison with the increase of internal friction. It is a common lecture-experiment to heat a bell by means of a burner placed underneath, and to demonstrate that all musical sound departs from the bell when struck provided the temperature approach a visible red. This absence of musical sound is generally, if not always, attributed to diminution of elasticity; but it would seem, from the above, to be much rather attributable to increase of internal friction†.

Another point to be noted in this experiment is the large permanent increase of internal friction produced by the rise of temperature. This doubtless arose from the fact that, though the mass of the suspended system only amounted to 40 grms., there was a sensible permanent lengthening of the wire resulting from the rise of temperature; for the author has shown that permanent extension and permanent torsion may both increase very considerably the value of the logarithmic decrement.

An equally important point to the above is the effect of rest on the logarithmic decrement when the wire has recently had

* A second set of experiments made with another piece of the same wire showed, however, a similar decrease of internal friction but at rather a lower temperature.

† The elasticity of iron at 1000° C. is nearly the same as the elasticity of silver at 20° C.

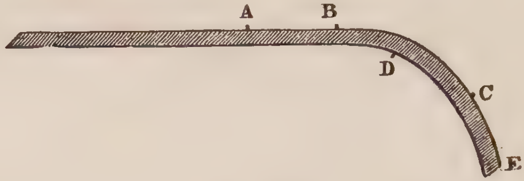
its temperature changed. It seems not improbable that, large as the internal friction is at the higher temperatures, it would be larger if tested *immediately* after the change had taken place. Again, it would be larger still if tested by a statical method; for the preliminary vibrations, to which the wire was subjected before testing, evidently reduced considerably the internal friction.

The Two Critical Temperatures of Iron.

The above experiment seems to show, not only that the internal friction of iron is very considerable at high temperatures, but also that there are two points, namely about 550° C. and 1000° C., at which there are very sudden changes in the rate at which the friction increases with the temperature. Now at or near these points there are, at any rate for annealed iron wire of good quality, very notable and sudden changes in certain of the physical properties of the metal. About 550° C. the metal begins to lose its magnetic properties very rapidly; so rapidly indeed that, if an iron wire be surrounded by a magnetizing solenoid always kept in action, and this again be concentric with a secondary solenoid connected with a galvanometer, a very sensible induced current can be observed at the critical temperature both on heating and cooling. It would seem likely also, from the researches of Prof. P. G. Tait and others, that at this temperature, or *about* this temperature, there are sudden changes in the thermoelectrical properties of the metal and also in the electrical resistance. These sudden changes would appear to indicate an equally sudden change in the molecular architecture of the iron; and it seems reasonable to suppose that when, on heating, the critical temperature is reached, a certain amount of the whole thermal energy which may be imparted by an electric current or a burner to the wire is used up in producing this change, whilst an equal amount of thermal energy is given out when the wire, on cooling, reassumes its original molecular structure. It by no means follows, however, that the so-called latent heat connected with the sudden change can be detected unless by very refined apparatus; for it is not at all unlikely that the sudden loss of magnetic properties may be due simply to a rotation of the molecules about their axes; and the researches of

the same temperature; but presently a delicate cloud appeared on the portion BC, which probably might have escaped notice

Fig. 2.



had it not been for the contrast caused by the unclouded portions AB, CE. Very shortly after its formation the cloud rolled away, and immediately afterwards a reglow occurred throughout the whole of the heated portion of the bar. The eyes had been kept upon the convex side of the bar in this trial; and as it was thought that the concave side might possibly show up bright instead of dark, by contrast with the unstrained metal, the bar was heated a second time and again watched on cooling. Rather contrary to expectation, the portion D, when the cooling had reached a certain point, showed up as being slightly clouded; as before, the cloud rolled away and the reglow immediately succeeded, which also, as before, seemed to raise the whole of the heated portion of the bar to the same temperature. The explanation of the formation of the cloud appears to be this: above but near the critical temperature the specific heat of the strained portion is lower than that of the unstrained portion, and, as a consequence, very shortly after the bar has been removed from the fire the temperature of the former is more rapidly lowered than that of the latter. Again, the reason why the strained portion has a less specific heat than the unstrained, and why, after the reglow, the strained and unstrained portions become apparently once more uniform in temperature, may well be that, in consequence of the greater internal friction of the strained portion, or from some other cause, the contraction of its molecules does not go on at so great a rate. When the temperature of a body is raised, part of the thermal energy expended on it is expended in producing *vis viva* of the molecules, and part in pushing them asunder. If we can prevent the molecules from being pushed asunder, a given amount of thermal energy imparted to the metal will

produce a greater rise of temperature. Similarly, when the metal is cooling, a given loss of thermal energy will be attended by a greater lowering of temperature when the contraction of the molecules is lessened. As a consequence, when the strained and unstrained portions are cooling near the critical temperature, there is a greater amount of molecular *potential* energy, and therefore a greater rise of temperature when the explosive change occurs at reglow, in the former than in the latter. The next experiment shows that effects similar to the above can be produced by hammering.

Experiment IV.—The straight portion of the bar used in Experiment III. was heated to a white heat, and whilst hot was placed upon a small anvil and struck twice very hard with the sharp end of a hammer at two separate parts, so that two rather deep dents were made in the bar about 10 centim. apart from each other. The bar was now heated a second time to a very bright red, and was allowed afterwards to cool. When a certain temperature had been reached a very beautiful phenomenon presented itself.

Suppose that AB and CD in fig. 3 represent the two dents.

Fig. 3.

Slight clouds were first seen at AB and CD ; these quickly deepened and extended both ways, leaving a bright space at Q. Immediately afterwards the clouds rolled away, beginning from Q, and the whole heated portion brightened with the recalescence. This phenomenon was so striking that the heating and cooling were repeated several times, with apparently no very rapid diminution of the effects of the strain. After the bar had been hammered and bent considerably in this and other experiments of like nature, several attempts were made after straightening it to anneal it ; these attempts were only partially successful. Always before recalescence faint patches of cloud could be detected here and there, showing that some portions were more strained than others. These clouds do not seem to be formed in those specimens of steel

so that, according to the above theory, there should be at least *two* points at which a sudden *reheating* takes place; but not necessarily two sensible *reglows**, or even, as we have seen, two *sensible* reheatings. This is all the more to be expected; because when, on heating, a wire under moderate stress has attained a temperature of about 550° C., there is evidently a sudden *permanent* yielding showing sudden softening of the metal, though this is not nearly so marked as the permanent yielding which takes place at the higher critical temperature. The next experiment *appeared* to show, not one recalescence, but several.

Experiment II.—The end of a steel poker was heated to a very bright red in a fire and was then taken to a corner of the room on one side of the fire. The room was quite dark save where it was faintly lighted up by the fire, and the eyes of the observer had been previously rested by shutting them for five minutes before the experiment. The heated part of the poker was intently watched whilst it was cooling and nothing was observed of any note for some little time. Presently, however, the external surface *appeared* to lose in temperature rapidly, and shortly afterwards to brighten most perceptibly. The apparent rapid darkening and reglow occurred no less than *seven* times during this same cooling. The experiment was repeated again and again, not only on the same night but on several nights, and as often as it was repeated *seemed* to show most conclusively that there were several reglows. It was noticed, however, that when the poker was loosely held in the hand the apparent reglows occurred more frequently than when it was fixed; and further that, when the outside of the poker was seen to darken, a slight motion of the poker in any direction caused it to brighten. Evidently convection-currents of air could not account for the phenomenon; for though a current of air might cause a sensible darkening of the external surface by its cooling effect, it could not cause the brightening which seemed to result from the motion. As it was thought that the brightening might be produced by the motion of the iron through the earth's magnetic field, the poker was reheated

* The reheating may take place at too low a temperature to produce a reglow.

and fixed in a horizontal position above the poles of a rather powerful electromagnet. As soon as one of the darkenings preliminary to a reglow occurred, the circuit of the battery which actuated the electromagnet was closed and then opened again; but there was no sensible brightening as a result of these operations, either at any point of this particular trial or at any point of any of several subsequent trials which were made. The next consideration was, Was the alternate darkening and reglowing a *subjective* sensation*? The poker was reheated, and after having been firmly fixed was watched whilst cooling. As soon as the preliminary darkening appeared, the head of the observer was turned slightly and instantly brought back again. The whole operation of turning the head away and back again certainly did not occupy more than a fraction of a second, and yet the metal shone out quite clearly again. Closing and opening the eyes as quickly as possible had a similar effect. No doubt such an instance of subjective sensation is well known to physiologists, but it has given the author a lesson which he will not readily forget respecting the danger of trusting to mere sensation†.

Several other attempts were made with bars and wires of iron and steel; and in the case of certain wires there seemed evidence of more than one real reglow. In face, however, of the warning given by the above experiment, the author prefers to leave the point still open until he has completed some calorimetric observations which he has in view.

Experiment III.—A flat iron bar, 60 centim. long, 1·5 centim. broad, and 2 millim. thick, was heated at one end to a very bright red, and observed when cooling in a dark room; a reglow occurred at a very high temperature‡. The bar was again heated, and whilst in the fire was bent as in fig. 2; it was then kept in the fire a short time longer, and afterwards removed. At first all the bar from A to E appeared to be of

* This was suggested by Mr. Burton at the Meeting of the Physical Society at which the paper was read; the author feels much indebted to Mr. Burton for the suggestion.

† Possibly the intensity of the apparent reglow will be found to differ with different individuals, and even to vary with the state of health at the time. To the author the phenomenon was very striking.

‡ Considerably above dull red.

Professor Ewing* prove that there may be considerable rotation of the molecules about their axes, and yet at the same time very minute change of temperature resulting from this rotation.

Near the higher temperature of 1000° C. there is again a most remarkable alteration in the behaviour of an annealed iron wire which is being heated when under stress or strain. If it is stretched by a slight weight it suddenly unstretches †; if it is under a slight bending-stress it suddenly unbends; and if it is under slight twisting-stress it suddenly untwists; whilst, on the contrary, if it has been previously bent or twisted permanently and then entirely released from stress, it suddenly bends more or twists more as the case may be ‡. Further, there are sudden changes, evidently at the same temperature, in the thermoelectrical relations which exist between strained and unstrained iron, and also between stressed and unstressed iron; these changes, like those before mentioned, being opposite in direction for stress and strain §. All these phenomena indicate, according to the author's belief, a second notable change in the molecular architecture of the iron; they certainly seem to indicate a change which must involve the expenditure of thermal energy to produce it when the iron is being heated, and the giving out of thermal energy when the iron is being cooled: in other words, here must be, one would think, a second point at which heat becomes latent.

Now suppose that the iron has been raised in temperature above 1000° C. and is cooling: when the critical temperature is reached the molecules would begin to come back to their original positions, just as water begins to freeze when the temperature of 0° C. has been reached; but internal friction prevents this, and it is not till the temperature has fallen, it may be very considerably, below the critical temperature that the readjustment takes place. When, however, the change under these conditions does take place it will be what Clerk Maxwell has designated as an explosive change, and the energy

* Phil. Trans. part ii. p. 553 (1885).

† Phil. Mag. [4] vol. xlvi. p. 472.

‡ *Ante*, p. 71. § *Ante*, p. 100.

set at liberty by the transformation will accelerate the subsequent rate of transformation. Here, again, we have an analogy in the case of freezing water; for in a Wollaston's cryophorus the temperature of the water may frequently be reduced several degrees below 0° C. before solidification sets in; but when it does, the whole mass, or a considerable portion of the whole mass, is frozen instantly. With both iron and water, therefore, there will be a greater amount of heat *suddenly* generated when the transformation takes place below the critical temperatures than when it occurs at the critical temperatures. Again, another analogy may be pointed out between ice at 0° C. and iron at 1000° C.; in both there is a marked change in the cohesion of the molecules. When an iron wire is under any but an exceedingly small stress of bending, torsion, or traction, it begins to yield permanently in a most astonishing manner when, on heating, the temperature reaches 1000° C.; the metal suddenly becomes, as it were, in a partially fluid condition.

According to the author's view, then, recalescence in cooling iron is owing to retardation, produced by internal friction, of a *physical* change somewhat resembling the change which occurs when water becomes ice: were it not for internal friction the change would take place comparatively in a gradual manner, and a certain amount of thermal energy would be equally gradually given out. In consequence, however, of internal friction the change is retarded until a temperature is reached below the temperature at which it would otherwise occur; when, partly owing to diminution of internal friction and partly to increased internal molecular stress, the molecules give way at some one point, and this is followed by a giving way throughout the whole mass; the change thus partakes of the nature of an explosion, and, as a consequence, there is a rapid rise of temperature. The following experiments were made with the object of collecting evidence for or against the above-mentioned theory.

Experiments on Recalescence.

It has been observed that there are *two* critical temperatures—one at about 550° C., and the other at about 1000° C.;

or iron which do not show the phenomenon of recalescence; at least so far as can be judged from observations made with one specimen of steel in which both clouds and perceptible reglow were entirely absent.

Experiment V.—Several attempts were made in this experiment to abolish recalescence by shaking or hammering. For this purpose a piece of pianoforte-steel wire, which showed recalescence well, was selected. The wire, which was 1 millim. thick, was suspended vertically with a weight of 1 kilo. on the end. The weight was held by one hand, whilst the other hand was employed in making the wire vibrate transversely whilst cooling; but no amount of vibrating could prevent the reglow from showing itself. A piece of the same wire was heated to a very bright red in a burner; it was then removed from the burner, immediately placed on an anvil, and a very sharp blow was given it by a hammer. The blow did not prevent the recalescence from appearing.

In this respect, therefore, the behaviour of iron which has cooled below 1000° C. without change of state seems to differ from water which has cooled below 0° C. without freezing; for the latter can be made to freeze by agitation. May not, however, the sudden freezing in this last case be due simply to the bringing of the liquid water into contact with small crystals of ice which have formed on the side of the vessel. Besides, the internal friction of the iron is very great above the temperature of 1000° C., and the vibrations may not have been made sufficiently near the point of recalescence*.

Experiment VI.—This experiment was made with a view of ascertaining whether the reglow would occur at a lower temperature the greater the amount of retentivity of the specimen. A specimen of well-annealed Swedish iron wire showed no trace of reglow. This may have been for two reasons:—In the first place, if the reglow occurs very near the critical temperature the very brightness of the wire renders detection difficult; in the second, the amount of potential energy *suddenly* converted into kinetic energy is less. The author does not believe that in this specimen there was no recalescence, for

* It was impossible to continue the agitations right up to the point of recalescence, as the motion prevented the observer from seeing whether the reglow took place or not.

the following reasons :—Whenever recalescence is very marked the sudden changes which have been mentioned as taking place in a wire cooling whilst under stress and strain are also well marked; and, conversely, if there is no sudden change in a wire under stress or strain there is no recalescence: the recalescence and the sudden changes seem to be inseparable companions. Now, with the particular specimen of iron in question, the sudden changes above alluded to could be detected; but they evidently occurred at a much higher temperature than with those specimens of iron and steel which showed recalescence plainly.

As far as could be ascertained from an examination of about a dozen different specimens of iron and steel, those specimens which seemed most capable of being softened by the process of annealing were those in which the phenomenon of recalescence was least marked. When recalescence is manifest, the temperature at which it appears seems to vary considerably with different specimens: with some this temperature would be at least as high as 800° C., with others at least as low as 550° C.

Views of Professor G. Forbes and Mr. H. F. Newall concerning Recalescence.

According to Professor Forbes*, the phenomenon of recalescence is due to the fact that at a certain temperature there is a sudden increase of the thermal conductivity of iron. As the metal cools from a white heat, the difference between the temperatures of the outside surface and of the inside will after a time become more or less considerable. The outside, being always cooler than the inside, will sooner reach the critical temperature at which there is a sudden increase of thermal conductivity, and, as a consequence, there will be a sudden rush of heat from the inside to the outside; hence the reglow. The assumption that a sudden increase of thermal conductivity should take place when the metal has cooled to a certain temperature is very reasonable; for there is, without doubt, a sudden increase (or, to speak more correctly, more than one† sudden increase) of electrical conductivity as the iron cools.

* Proc. R. S. E. April 6, 1874.

† We should expect, therefore, more than one surface-*reheating*

Mr. H. F. Newall asserts *, however, that he has shown that the reglow is "not due to differences in conductivity in iron at different temperatures," and that "there is a rise of temperature not only at the surface . . . but also throughout the mass." Mr. Newall has not yet brought before us any experimental evidence in proof of these assertions; but he has promised to do so.

Mr. Newall seems to agree with the author that the process going on during recalescence "partakes of the nature of an explosion, in that once started it continues throughout the mass of the iron," but he regards the rise of temperature as being caused by internal *chemical* action. The author looks forward with much interest to the publication of Mr. Newall's results.

XVII. *On the Price of the Factor of Safety in the Materials for Lightning-rods. By* Professor SILVANUS P. THOMPSON, *D.Sc.*†

IT is possible to determine in an absolute manner what metal is best for securing safety by a system of lightning-conductors, for a given prime cost. In the calculations that follow it is assumed that the duration of the lightning-discharge is so brief that there is no time for appreciable radiation or convection of heat from the surface of the conductor, or for conduction into other bodies; and that the conductor is devoid of self-induction.

Safety is dependent, other things being equal, upon the difficulty of fusion of the conductor. The difficulty of fusion of the conductor varies directly as the fusing-point of the material (or, more strictly, as the difference between this temperature and that of the surrounding air, assumed here to be 15° C.), and inversely as the temperature-rise occasioned in it by the discharge of electricity, the amount of which is supposed to be given. The temperature-rise varies inversely as the specific thermal capacity of the material, and directly as the heat developed in it. The heat caused by a given discharge of electricity varies directly as the total electric resistance of the

* Phil. Mag. vol. xxiv. No. 150, p. 436 (1887).

† Read January 28, 1888.

conductor. The total electric resistance of the conductor varies directly as the specific electric resistance per unit cube of volume of the material of the conductor, also directly as its length, and inversely as its area of cross section (supposed uniform throughout). The area of section varies directly as the volume and inversely as the length (supposed given). The volume varies inversely as the density of the material and directly as its mass. The mass of the conductor varies directly as the total cost, and inversely as the cost per unit of mass.

Hence, writing f for the temperature of fusion above that of the surrounding air (assumed at 15° C.), s for the specific thermal capacity, ρ for the specific electric resistance, l for the given length, d for the density, k for the cost, in pence, per pound of the material of the conductor, it at once follows that

$$\text{Safety varies as total cost} \times \frac{f \times s}{\rho \times d \times l^2 \times k}.$$

Hence, as the total cost and the length are supposed to be given, we have

$$\text{Factor of safety for equal total cost} = \frac{f \times s}{\rho \times d \times k}.$$

The only metals about which it is necessary to inquire are copper, silver, iron, lead, platinum, and tin. In the following table the values of f are deduced from the figures of Violle and of Rudberg. The values of k are deduced from the market-prices in London on December 7, 1887 : viz. copper, £75 per ton ; silver, 4*s.* per troy ounce ; iron (rod) £5 10*s.* per ton; lead, £16 10*s.* per ton; platinum, 32*s.* per troy ounce ; tin, £170 per ton. The figures given in column 6 for the factor of safety for equal cost are calculated by the foregoing formula multiplied by 10^4. The figures in the seventh column, giving the total cost of equal safety, so far as prime cost of material is concerned, are of course inversely proportional to the figures in column 6.

	f.	s.	ρ.	d.	k.	Factor of safety.	Total cost for equal safety.
Copper ...	1039	0·0949	1615	8·94	8·045	9·488	£100
Silver	939	0·0570	1609	10·51	699·84	0·045	£18,770
Iron	1585	0·1138	9827	7·79	0·589	39·97	£21 9*s.*
Lead	307	0·0314	19847	11·35	1·769	0·242	£3508
Platinum..	1760	0·0325	9158	21·36	5598·7	0·00052	£1,621,600
Tin	213·5	0·0548	13360	7·29	18·22	0·6593	£1287

The prime cost of copper is known to be greater than that of iron for equal conductivity; and reckoning on this basis, irrespective of difficulty of fusion, it is stated in the Report of the Lightning-rod Conference to be 50 per cent. dearer. If that were all, then doubtless the small saving in using iron would be more than counterbalanced by the attendant disadvantages of deterioration by rust, and the like. But when the higher fusion-point, the greater capacity for heat, and the lesser density of iron are taken into account, the superiority of iron, for equal total prime cost, becomes so marked as to once more raise the question whether it may not be expedient to return to the original practice of Franklin, and use lightning-rods of iron instead of those of copper.

London, December 12, 1887.

XVIII. *An Experimental Study on the Influence of Magnetism and Temperature on the Electrical Resistance of Bismuth and its Alloys with Lead and Tin. By* Edmond von Aubel, *of Liége, Member of the Physical Society of London.**

Preliminary Communication.

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Many physicists have been at work upon the influence of

* Read January 28, 1888.

magnetism on the electrical conductivity of metals. The metals which have already been studied are iron, nickel, cobalt, antimony, tellurium, and especially bismuth.

The above investigations have been carried on principally with the view of discovering an explanation of Hall's phenomenon.

The present memoir is intended to fix the date of publication. Our researches were commenced last May ; since then many works have appeared on the question which we were studying, notably the excellent memoirs of Messrs. Goldhammer*, A. von Ettingshausen†, and W. Nernst‡.

Quite recently we saw, in the *Wiener Anzeiger*, 1887, p. 222, that Messrs. von Ettingshausen and Nernst had made a communication to the Academy of Sciences at Vienna, having the title "Ueber das thermische und galvanische Verhalten einiger Wismuth-Zinn-Legirungen im magnetischen Felde." This paper is not yet published, as far as we know ; so that we cannot take account of it in what follows.

Our researches are far from being finished. They will be continued as soon as our other occupations will permit.

I. *The Preparation of Rods of Bismuth.*

We have studied bismuth under three molecular states :—

(1) Melted and cooled slowly,

(2) Melted and rapidly cooled or tempered,

(3) Compressed.

Let us examine the preparation of rods of bismuth in these three cases.

1. In order to prepare bismuth wires slowly cooled, capillary glass tubes were procured, to which were joined at both ends, at right angles to the capillary part, glass tubes of fairly large diameter. These capillary tubes were heated in a sand-bath, and some grains of bismuth were introduced into one of the side branches. When the metal was melted, it ran down by its own weight into the capillary tube. The

* We have given in the Bibliography only the list of works relating to bismuth.

† *Vide* the Bibliography.

‡ *Annalen der Physik*, 1887, No. 8*b*.

capillary tubes were then left to cool very slowly; they were taken from the sand-bath only when it was quite cold. It often happens that, owing to the unequal expansions produced during the cooling of the bismuth and the glass, the tubes are broken; for this reason we used capillary tubes with very thin walls.

Working in this way, any formation of oxide in the capillary part is entirely avoided; and this part alone must be used in the experiments.

We think that this way of preparing bismuth wires is much more convenient than the method by exhaustion employed by others*, and which cannot furnish results entirely comparable, because there is no certainty that the melted metal is always cooled under the same conditions. The bismuth rod being thus prepared, the two side branches which are not wanted are taken away.

The electrodes to convey the electric current remain to be fixed. For this purpose one end of the capillary rod of bismuth is heated at the same time as a platinum wire in a Bunsen burner up to the melting-point of bismuth; then the red-hot platinum wire is plunged into the capillary tube. It is easy to see that a good contact is thus produced.

2. In order to obtain bismuth rods very rapidly cooled or tempered, the melted metal was poured rapidly into a kind of iron trough, cold, and forming a somewhat acute dihedral angle. If the pouring takes place rapidly, by inclining the trough sufficiently to cause the melted bismuth to run down very quickly, rods of bismuth can be obtained sufficiently long and not too thick. This last condition is indispensable, otherwise the bismuth rods will have too small a resistance.

No adherence takes place during this tempering between the iron and the bismuth. Moreover, by taking certain precautions, the surface of the bismuth may also be filed, and a fresh part of the metal be thus exposed.

In this case the electrodes were made of copper wire, soldered to the two ends of the rod of bismuth by means of a fusible alloy.

* See notably the memoir of Leduc, *Journal de Physique* [2], iii. p. 363.

3. The compressed bismuth which we used in our experiments was given to us by Prof. Spring. Having been prepared in a drawplate*, it was absolutely cylindrical.

The electrodes were formed, as in the case of the tempered bismuth, of copper wires soldered to the bismuth with a fusible alloy.

The molecular arrangement in this compressed bismuth is very remarkable. A fracture normal to the axis of the cylinder presents a radial structure. We shall see later on the curious results that this sample has given us.

The alloys have only been studied under two states :—

(1) Melted and very slowly cooled,

(2) Tempered.

Rods of these alloys have been prepared in the same way as those of bismuth.

II. *Alloys.*

To obtain the alloys, two sandstone crucibles are used. Bismuth is put into one, tin or lead into the other. These metals have previously been weighed. When the bismuth is melted it is kept liquid, being warmed gently to avoid too much oxidation ; it is then poured upon the other metal, which is also liquid. The contents of the crucible are then poured into the other, and this decanting is repeated several times. We have worked in this way in order to obtain alloys in which the proportion of tin or lead should be given almost exactly by the weight, and to avoid having to make analyses of the alloys in order to know their approximate composition.

III. *Analyses of the different Specimens of Bismuth studied.*

We have measured the resistance of several specimens of bismuth. They came from the manufactory of chemical products of Messrs. Monheim at Aix-la-Chapelle, Trommsdorff at Erfurt, and Schering at Berlin. Mr. Monheim's bismuth

* *Annales de la Société Géologique de Bélgique*, t. xi. p. cxxxiv (1884).

is commercially pure metal. We have three different specimens from the firm of Trommsdorff, which we shall call, for brevity, "Trommsdorff I.," "Trommsdorff II.," and "absolutely pure Trommsdorff."

The two first, as well as the bismuth from the Schering factory, are products referred to as very pure in the catalogue of manufacturers.

The "absolutely pure Trommsdorff" bismuth, of which the cost was far higher than that of the others, has been specially prepared for us in the Erfurt manufactory.

As to the compressed bismuth, it has been obtained by means of the commercial metal. Prof. Spring dissolves this metal in HNO_3, precipitates it by pouring the solution into a large quantity of water, filters, and washes it. He then redissolves the basic nitrate, precipitates a second time, calcines, and reduces in a current of pure hydrogen.

We give all these directions, so that it may be possible for physicists to repeat our experiments, and in order to show how little confidence ought to be placed in certain determinations which have been made on the physical constants of bismuth.

M. A. Classen, Professor of Analytical Chemistry at the Upper Technical School at Aix-la-Chapelle, was so kind as to undertake to make qualitative analyses of various specimens for us. We wish again to thank him here for the indispensable assistance which he has given to our work.

Results of the qualitative analyses :—

Bismuth.	Impurities.
Monheim . . .	*Copper**, *lead*, iron.
Trommsdorff I. .	*Iron*, copper.
Trommsdorff II. .	*Lead*, copper, nickel.
Absolutely pure Trommsdorff .	*Lead*, copper (traces), *iron*, carbon.
Schering . . .	Copper, lead ?, nickel.

It is very difficult to prepare pure bismuth. Even the method indicated above for preparing the metal which has

* The metals whose names are in italic are those which were found in considerable quantities in the bismuth analysed.

been compressed, does not yield a pure product. A small quantity of lead and copper is always precipitated with the bismuth; and it is only by repeating this operation several times that the metal can be obtained very pure.

M. A. Classen is occupied at the present time in preparing for us pure bismuth, which is to be used in our later researches.

It results from all this that the various specimens of so-called pure bismuth are very different from a chemical point of view. Their points of fusion and their specific weights have been likewise determined; they vary very sensibly from one sample to another.

IV. *Measurement of Electrical Resistances.*

The threads of bismuth were placed in a water-bath, which was heated all along its length by a linear gas-burner, forming a series of small flames: a thermometer indicated the temperature.

The whole was placed between the poles of a Ruhmkorff's electromagnet of large size, worked by an Otto gas-engine and a dynamo-electric machine of Siemens and Halske*.

The intensity of the electric current in the spirals of the electromagnet was sensibly 28 amperes during the whole course of the experiments.

The ordinary poles of the electromagnet were replaced by large flat poles, made of thick iron circular plates, each one having a thickness of 15 millimetres and a diameter of 150 millimetres. In this way the magnetic field was made more uniform.

In order to measure the resistances, Thomson's method was employed, and a Siemens's *dead-beat* galvanometer, with the magnet in the form of a bell, was used.

In order to avoid heating the thread of bismuth by the passage of the current, one Grove cell only was used, and, by the aid of a see-saw commutator, the current was not allowed

* The electromagnet was set in such a direction as to have no influence on the galvanometer.

to pass in the bismuth for a longer time than was necessary for the observations.

The deviations of the galvanometer-mirror were observed in a telescope furnished with a graduated scale, the distance between each division being two millimetres, and placed at a distance of more than seven metres from the mirror. One can judge of the sensitiveness of the method. We have also employed Kirchhoff's method of measuring resistances, preserving all the other arrangements. A great number of rods of bismuth and its alloys have been tried ; in the following tables we only give the values obtained for a sample of each kind.

In our experiments we did not obtain directly the resistance of the thread of bismuth, because in Thomson's method the wires for conveying the current were not fixed directly to this metal, but to the copper or platinum conductors which served as electrodes. It was necessary therefore to deduct the resistances of these copper or platinum conductors, in order to obtain values relating only to the bismuth. In the case of the wires slowly cooled, we have subtracted the values obtained directly, as the resistance of the platinum conductors exterior to the wire of bismuth, because this latter metal adheres very easily to platinum, and the section of the rod of bismuth is very great when compared with that of the thread of platinum.

V. *Results of Electrical Measurements.*

The values of W are the electrical resistances in Siemens's units ; the values of Wm are the electrical resistances in the same units under the action of a magnet.

(a) *Rods of Bismuth slowly cooled.*	Temp.	W.	Wm.
Trommsdorff bismuth, absolutely pure.........	17°·2	0·4100	0·4120
	42·7	0·4050	0·4070
	70·8	0·3953	0·3966
Trommsdorff bismuth I.	20·4	0·1000	0·1010
	42·4	0·1092	0·1099
	69·9	0·1210	0·1217
Monheim bismuth	18·6	1·1295	1·1455
	41·2	1·0425	1·0525
	72·2	0·9375	0·9435
Schering bismuth.........	18·2	0·2522	0·2572
	43·5	0·2652	0·2662
	71	0·2789	0·2789
Trommsdorff bismuth II.	20·4	0·1661	0·1671
	43	0·1713	
	73	0·1811	0·1821

(b) *Rods of Bismuth tempered.*	Temp.	W.	Wm.
Trommsdorff bismuth, absolutely pure.........	15·2	0·1788	0·1790
	40·7	0·1805	
	69·7	0·1812	0·1812
Monheim bismuth	18·4	0·0934	0·0940
	43	0·0932	0·0937
	70	0·0909	0·0911
Trommsdorff bismuth II.	16·2	0·0784	0·0789
	40·8	0·0774	0·0777
	74·2	0·0765	0·0770
Schering bismuth	20·2	0·1239	0·1254
	42	0·1239	0·1249
	72·4	0·1244	0·1249
Trommsdorff bismuth I.	18·2	0·0510	0·0515
	41·6	0·0512	0·0517
	71·4	0·0541	0·0545

(c) *Alloys with Tin.* (1) Rods slowly cooled.	Temp.	W.	Wm.
Absolutely pure Trommsdorff bismuth. 0·39 gr. tin to 100 gr. Bi.	19	0·3364	0·3374
	42·8	0·3444	0·3454
	71·6	0·3475	0·3475
Trommsdorff bismuth I. 0·46 gr. tin to 100 gr. Bi.	19·6	0·3966	0·3976
	45·2	0·4066	0·4066
	70·6	0·4166	0·4166
Schering bismuth. 0·40 gr. tin to 100 gr. Bi.	19·8	0·3643	0·3663
	41·2	0·3733	0·3743
	69·8	0·3803	0·3813
Trommsdorff bismuth II. 0·46 gr. tin to 100 gr. Bi.	21	0·5133	0·5183
	42·6	0·5258	0·5288
	71·8	0·5343	0·5363

Table (*continued*).

(c) *Alloys with Tin* (cont.).

(2) Rods tempered.

	Temp.	W,	W*m*.
Trommsdorff bismuth, absolutely pure. 0·39 gr. tin to 100 gr. Bi.	18·6 41·6 70·4	0·2133 0·2173 0·2213	0·2136 0·2178 0·2213
Schering bismuth. 0·40 gr. tin to 100 gr. Bi.	16·6 41·8 73·9	0·2100 0·2145 0·2148	0·2105 0·2150 0·2150
Trommsdorff bismuth I. 0·46 gr. tin to 100 gr. Bi.	20·4 42·8 69·6	0·1443 0·1478 0·1498	0·1443 0·1478 0·1498
Trommsdorff bismuth II. 0·46 gr. tin to 100 gr. Bi.	24·5 40·6 77·8	0·1529 0·1554 0·1574	0·1534 0·1561 0·1574

(d) *Alloys with Lead.*

(1) Rods slowly cooled.

	Temp.	W,	W*m*.
Trommsdorff bismuth, absolutely pure. 0·52 gr. Pb to 100 gr. Bi.	18·8 44·7 68·9	0·4925 0·4995 0 5015	0·4945 0·5015 0·5035
Trommsdorff bismuth I. 0·54 gr. Pb to 100 gr. Bi.	21 43·4 71·6	0·2713 0·2778 0·2878	0·2743 0·2788 0·2888
Schering bismuth. 0·49 gr. Pb to 100 gr. Bi.	18·8 44·8 67·4	0·2631 0·2611 0·2620	0·2656 0·2632 0·2630
Trommsdorff bismuth II. 0·60 gr. Pb to 100 gr. Bi.	15·4 43 69·8	0·3923 0·3878 0·3848	0·3963 0·3913 0·3871

(2) Rods tempered.

	Temp.	W,	W*m*.
Trommsdorff bismuth, absolutely pure. 0·52 gr. Pb to 100 gr. Bi.	18·2 42·4 70·4	0·1690 0·1725 0·1735	0·1700 0·1730 0·1735
Trommsdorff bismuth I. 0·54 gr. Pb to 100 gr. Bi.	17·4 41·6 71	0·0937 0·0929 0·0914	0·0947 0·0934 0·0919
Schering bismuth. 0·49 gr. Pb to 100 gr. Bi.	18·8 44·4 70·2	0·1478 0·1458 0·1439	0·1493 0·1468 0·1449
Trommsdorff bismuth II. 0·60 gr. Pb to 100 gr. Bi.	22·2 43·6 72·2	0·1056 0·1056 0·1031	0·1066 0·1061 0·1031

Table (*continued*).

(e) *Compressed Bismuth.* (From Prof. Spring.)

	Temp.	W.	Wm.
Compressed bismuth ...	16·8 42·4 76	0·1148 0·1140 0·1150	0·1157 0·1145 0·1153
The same compressed bismuth, melted, cooled very slowly in a sand-bath, and then operated upon again.	16·2 42·3 71·7	0·1861 0·2051 0·2263	0·1871 0·2053 0·2263

VI. *Inferences drawn from the preceding Tables.*

1. *The Influence of Temperature.*—If the preceding results be examined, it is easy to see at once that the different samples of bismuth examined behave quite differently. Some of them give an increased resistance, and others, which is remarkable, a diminished resistance, when the temperature rises. This last fact, noticed for the first time by M. Righi, is not due, according to this physicist, to the presence of arsenic as an impurity in the bismuth, but ought to be attributed to the tin, which, even in very small quantities, is able to produce this result.

A number of experiments have been made by us to discover the reason of this diminution of resistance. We studied first alloys of bismuth and tin, in which the latter metal was present in comparatively large quantities. On examining Table (c), it will be seen that these alloys always give an increase of resistance with the temperature, while the bismuth which enters into their composition produces a diminution of resistance for a rise of temperature. Besides, the chemical analysis has shown us that not one of our samples of bismuth contains any tin as an impurity. It is then sufficiently demonstrated that it is not the metal that is the cause of the observed anomaly*. A glance at the results of the chemical analyses and at the electrical measurements will equally prove

* See also the memoir of Mr. W. Nernst (*Annalen der Physik*, 1887, p. 783), which has appeared during the course of our researches.

that the reason for the fact ought not to be looked for in the presence, as impurities, either of arsenic or of iron.

Mr. C. W. Kayser* has shown that expansion on solidification has not taken place when bismuth is combined with 10 per cent. of lead.

For this reason we have examined also the alloys of bismuth and lead, which have furnished us with very interesting results†, recorded in Table (d), but which do not in any way explain the diminution of resistance when the temperature rises.

Some influence might still be attributed to the capillary glass tube in which the wire of bismuth is placed. On heating, the glass and bismuth expand differently, and mutually impede each other in their movements. We have therefore successively studied a rod of bismuth cast in a capillary tube, and the same rod after having broken the glass (with precautions) in such a way as not to alter the metallic wire; no change was observed in the direction of these phenomena. Besides, the rods of tempered bismuth were not prepared in glass tubes, and yet some of these present the same peculiarity.

We likewise assured ourselves that there was no relation between the abnormal electrical phenomenon and the points of fusion or specific gravities of the different kinds of bismuth‡.

2. *Influence of Magnetism.*—On the whole the influence of magnetism on the resistance of bismuth has been more feeble than that which has been announced by M. Righi §. Magnetism always produces an increase of resistance; its influence diminishes when the temperature rises, and is more feeble in the alloys than in the bismuth itself.

* *Berichte der deutsch. chem. Gesellsch. Berlin*, 1869, p. 309.

† M. Leduc had already studied the conductility of the alloys of lead and bismuth (*Journal de Physique* [2], v. p. 116), but from another point of view.

‡ Mr. C. L. Weber has made known, in 1886 (*Annalen der Physik* xxvii. p. 145), some very curious results on the influence of temperature on the electrical conductility of the alloys of Rose, Wood, and Lipowitz; but this work has no immediate connexion with our study.

§ Mr. Goldhammer (*l. c.*) has also found the influence of magnetism more feeble than that indicated by M. Righi.

3. *Study of Compressed Bismuth.*—The resistance of compressed bismuth hardly varies with the temperature; from 16°·8 to 42°·4 there was a slight diminution; then an increase up to 76°, also very small.

After these first experiments, we took away the two ends of the threads of compressed bismuth, on which was the fusible alloy. With the rest we prepared a rod of bismuth slowly cooled by the method indicated above.

Measurements of the electrical resistances of this bismuth, in this state, gave a fairly large increase of resistance when the temperature rose.

There is, then, a notable difference, with regard to the molecular structure, between bismuth prepared in a drawplate and that which, having been melted, has been allowed to cool slowly. One may conceive in this latter case that the crystals take up their own positions during the cooling, whilst the metal prepared in a drawplate is formed of parallel fibres of bismuth.

More complete electrical researches might make clear this important question of molecular physics. One might especially compare bismuth, compressed in an ordinary compressor, with that prepared in a drawplate.

VII. *Conclusions.*

To sum up, the diminution of electrical resistance observed in certain kinds of bismuth and in some alloys with lead cannot as yet be explained. The molecular structure, which we have modified by tempering and compression, has a great influence on the action of the temperature.

We hope that our further researches will lead us to an explanation of the anomalies which we have pointed out.

In the course of this paper we have never mentioned the name of Prof. Ad. Wüllner, our learned and venerated master; we might well have done so at each step. We are glad to be able here to express our thanks to him for the facilities which he has procured for us for undertaking our researches.

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NOTE.

M. G. P. Grimaldi has recently * studied the influence of magnetism on the thermoelectric properties of bismuth. He denotes by ϵ the thermoelectric electromotive force of the bismuth-copper couple, when it is out of the magnetic field; by ϵ' the electromotive force of the same couple in the magnetic field; and takes for a measure of the phenomenon the quantity $\delta = \frac{\epsilon - \epsilon'}{\epsilon}$.

This physicist finds that δ is positive for commercial bismuth and negative for pure bismuth.

One ought, perhaps, to compare this result with those we have obtained relative to the influence of temperature on the electrical resistance of different kinds of bismuth.—EDM. VON AUBEL.

* *Journal de Physique*, Dec. 1887, p. 569. This paper is a *résumé* written by the author of a preliminary note presented to the R. Accademia dei Lincei (Feb. 7, 1886) and of a memoir presented in June 1886 to the Societa di Scienza naturali ed economiche di Palermo.

See also *Beiblätter zu den Annalen der Physik*, 1887, no. 6, p. 472.